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What is a fractal binary pattern?
By converting each positive integer including zero into its respective sequence of vertically-oriented binary digits (1 is grey and 0 is white), we get an infinite visual image with self-similarity properties or a fractal binary pattern.

What is the Box Counting Method or BCM for short?
It is a well-known technique used to extract numerical information from an image. Such information is called BCM data points and are used to calculate the fractal dimension of an object. The box counting method was applied to our binary pattern and after carrying out all calculations we obtain a numerical value between one and two thus demonstrating that our visual image is indeed a fractal binary pattern.

What is meant by "mathematical background"?
In physics, the cosmic background radiation is defined as the remnant from the Big Bang. To put it another way, the cosmic background radiation is the physical background for all and any activity since the beginning of time itself. On the other hand, I have presented in my book the concept of mathematical background as the abstract framework underlying any mathematical object. If we assume that mathematics deals with four basic notions: quantity, space, change and structure. The number line is the mathematical background for quantity since with it we can define the natural numbers, the integers and the real numbers.

Why is the distribution of prime numbers hidden within a fractal binary pattern?
The mathematical background for the prime numbers involves all of the four previously mentioned notions. Three branches of mathematics converge to reveal the pattern behind the distribution of the prime numbers. These are fractal geometry, analysis and Boolean algebra. Fractal geometry is a property of space, analysis deals with change and Boolean algebra with structure and quantity. If we classify the prime numbers into three main categories: consecutive, twin and single. Consecutive primes are pairs of primes which differ by one and have the form $(p, p+1)$, where p is a prime such as $(2, 3)$. Twin primes have the form $(p, p+2)$ such as $(11, 13)$. Single primes are isolated primes that differ from their nearest prime by at least four. The first single prime is 23 since $23 - 4 = 19$ and $23 + 6 = 29$, which are both prime numbers, as expected. Under this classification scheme each prime number can belong to one and only one of these categories. On a closer look, it is easy to prove that our infinite visible binary pattern is continuously subdivided into self-similar structures and that all twin primes (whether finite or infinite in number) are exactly confined within these structures. As a result, the distribution of primes numbers obeys a fractal binary pattern.

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What are p-adic numbers?
P-adic numbers are the creation of the German mathematician Kurt Hensel $(1861 - 1941)$ and have found many applications beyond mathematics. Such numbers can be defined as

$$|N|_p = |Q \times p^n| = \left(\frac{1}{p}\right)^n$$

where N is a rational number, Q is the quotient of two integers, p is a prime and n is an integer. In everyday language, p-adic numbers provide a new way to measure the "distance" between two rational numbers. Since our universe was infinitesimal at the very beginning, p-adic numbers play a major role within theoretical physics.

What is a p-adic universe?
A p-adic universe where p stands for prime number is a mathematical model that tries to describe our universe at the Planck length where quantum physics unites with gravitation.

How is the Collatz problem connected to the prime numbers?
Assume that one is also a prime number. Thus, for every positive integer N such that $f(N) = 3N + 1$ its output must satisfy either $\frac{3^a N}{2^b} + \frac{P}{2^c} = 1$ where P is either a prime or a product of two or more primes, or $\frac{N}{2^d} = 1$ where a, b, c and d are positive integers not necessarily identical. For example, if $N = 7$ then its corresponding equation is $\frac{243N+347}{2^{11}} = 1$ or $\frac{3^5 \times 7}{2^{11}} + \frac{347}{2^{11}} = 1$ where 347 is a prime. In the case of $N = 16$ we get $\frac{N}{2^d} = 1$ or $\frac{16}{2^4} = 1$. Hence, if you look at the integers present in the previous general equations, it is easy to note that the Collatz problem is directly connected to the primes 1, 2 and 3.

Is 3x + 1 the only function that works as a solution to the Collatz problem?
Surprisingly, the answer is no. Consider the general function $f(x) = ax + b$ where a and b are positive integers. If both a and b are identical powers of two then all of the respective functions $f(x) = 2^k x + 2^k$ where k is bigger or equal to zero satisfy the Collatz problem. For example, the first such function is $f(x) = x + 1$ where $k = 0$ and for $N = 5$ it is easy to calculate the resulting sequence as $5 \rightarrow 6 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$. Likewise, for $k = 1$ we obtain that $f(x) = 2x + 2$ and, once again, for $N = 5$ the respective sequence is $5 \rightarrow 12 \rightarrow 6 \rightarrow 3 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ as expected. You can prove that for any identical powers of two the previous function will work with every positive integer, that is, the final number of each corresponding sequence will always be one.

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What are probabilistic primality tests?

Probabilistic primality tests are mathematical techniques that establish that a given number is prime with a certain degree of error. In other words, such tests cannot fully guarantee that the number under analysis will always be prime. Once such test is known as Fermat's Little Theorem. It states that if p is a prime greater than three then $k^p - k$ is divisible by p for all k equal or greater than two and equal or smaller than p minus two where k is a positive integer. For example, $2^{137} - 2$ and $3^{137} - 3$ along with $4^{137} - 4$, etc., until $135^{137} - 135$ are all divisible by 137; consequently 137 is a prime number. Unfortunately, many integers satisfy these conditions but are not prime.

What are deterministic primality tests?

Deterministic primality tests are mathematical techniques that determine with absolute certainty that a given number is prime. One such test is the Lucas-Lehmer primality test for Mersenne primes. Recall that Mersenne primes are numbers of the form $M_p = 2^p - 1$ where p is a prime. The test makes use of the Lucas sequence and the operation of modular arithmetic. The Lucas sequence L_i is a sequence of numbers obtained by taking the previous integer, squaring it and subtracting two from the result. This sequence starts with the integer 4, and the numbers generated are 14, 194, 37634 and so on. Finally, the test states that M_p is a Mersenne prime if and only if $L_i \; mod \; M_p = 0$ for i equals p minus two where p is a prime number. For example, in order to test the primality of $2^5 - 1 = 31$ observe that $p = 5$ then $i = p - 2 = 3$. Please note that $L_0 = 4$ and therefore $L_2 = 194$. Next, calculate L_3 and compute $L_3 \; mod \; 31$. It is to realize that $37634 \; mod \; 31 = 0$ because $37634 = 1214 \times 31$ so $2^5 - 1$ is prime!

What are mirror primes?

Mirror primes are primes of the form $2^{2^n} - 3^m$ and $3^{3^n} - 2^m$ where n and m are both integers. For example, $2^{2^3} - 3^3 = 229$ and $3^{3^2} - 2^8 = 19427$. They are called mirror primes because you can exchange every two for a three and vice versa in any of the general equations above.

Can Fermat numbers be defined in terms of primes?

Yes, they can. Every Fermat number $F(n)$ can be represented as $F(n) = 2^{2^n} + 1 = p + 3^m + 1$, where both n and m are integers and p is a prime. For example, $F(3) = 2^{2^3} + 1 = 257$ can also be written in two different ways or $257 = p + 3^m + 1$ where $p = 13$ and $m = 5$ or $p = 229$ and $m = 3$.

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Can the first three odd primes be used to generate other primes?
Yes, they can. For certain even values of n the function $3^n + 5^n + 7^n$ is prime. For example, for $n = 14$ we obtain that $3^{14} + 5^{14} + 7^{14} = 684331371443$ which is a prime. The next primes are generated when $n = 24$; $n = 26$ and so on.

Can the first four primes be used as a primality test?
Yes, they can. If n is a prime bigger than two then $2^n + 3^n + 5^n + 7^n \; mod \; 6n = 17$, otherwise n is composite. For example, consider $n = 3$ which we know to be prime then $2^3 + 3^3 + 5^3 + 7^3 = 503$ and $503 \; mod \; (6 \times 3) = 17$ as expected. On the other hand, consider a small integer as $n = 9$, we get $2^9 + 3^9 + 5^9 + 7^9 = 42326927$ and $42326927 \; mod \; (6 \times 9) = 53$. In conclusion, our primality test has correctly identified nine as being a composite number.

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