

## Professionals

### What is meant by an “experimental observation”?

In my book, I have used the term “experimental observation” to describe any particular mathematical pattern discovered and supported by extensive computational evidence. For example, my first experimental observation concerns the possibility of transforming a composite number into a prime by changing a single decimal digit of the given number.

### Can we transform all the RSA challenge numbers into corresponding primes?

No, although two different experimental observations were applied to all the RSA challenge numbers we do not obtain a prime in every case. There are eight RSA challenge numbers, but only seven of them can be transformed into primes. The exception is RSA-896 where 896 stands for the number of binary bits. In other words, RSA-896 represents a 270 decimal digit number which has not been factored to date.

### How can transforming Fermat numbers into primes help you to factor them?

My third experimental observation states that if you obtain a prime by subtracting some power of ten from a given Fermat number, then the decimal size of one of its prime factors must be either equal to that power of ten or one plus that power of ten. Furthermore, please note the possibility that a certain Fermat number can be transformed into a prime but no corresponding prime factors exist for such number. Nevertheless, this observation provides (when applicable) an indirect way to uncover the decimal size of one of the prime factors of any Fermat number without actually attempting to factor the number itself.

### Within the fractal binary pattern what is meant by “key”?

As mentioned previously the distribution of the prime numbers fits a fractal binary pattern. This means that our visible binary pattern is continuously subdivided into self-similar structures which are called keys. Each of these structures represents all the grey and white colored squares corresponding to the natural numbers between (and including)  $2^k$  and  $2^{k+1} - 1$  where the positive integer  $k$  indicates the respective key. For example, the first key or  $k = 1$  contains the first two prime numbers, that is, two and three. Likewise, the second key or  $k = 2$  contains all the natural numbers between 4 and 7. These are four, five, six and seven. All successive keys are generated in this manner.

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**Can we mathematically prove that twin primes are confined within the key structures?**

Yes, we can. The starting point of our proof is the following:

$$(last\ integer\ of\ any\ key, second\ integer\ of\ next\ key) = (2^n - 1, 2^n + 1)$$

where  $n$  is a positive integer. Remember that twin primes have the form  $(p, p + 2)$  where  $p$  is a prime. In fact, all twin primes except for the pair  $(3, 5)$  can be calculated via the simple well-known equation  $(p, p + 2) = (6v - 1, 6v + 1)$  where  $v = 1, 2, 3$ , and so on. At this point, we equate the previous formula with that above and get  $(2^n - 1, 2^n + 1) = (6v - 1, 6v + 1)$ . If we equate their respective left-hand sides then  $2^n - 1 = 6v - 1$  and this can only be true if  $n = p$  or a prime number. However, in such case, we get that  $2^n - 1 = 2^p - 1$  which is a very famous type of prime known as Mersenne prime. Finally,  $2^p - 1 = 6v - 1$  and solving for  $v$  we obtain that  $v = \frac{2^{p-1}}{3}$  therefore  $v$  is not an integer. Mathematically speaking this means that all twin primes (whether finite or infinite) are confined within the previously mentioned key structures and the proof is done.

**Can we prove that every twin prime is either a pair of an ordinary prime and a Gaussian prime or vice versa?**

Yes, we can. The converse of the above means that there is no twin prime where both numbers are either ordinary primes or Gaussian primes. To be more precise, our definition for the twin primes has become  $(p, p + 2) = (OP, GP)$  or  $(GP, OP)$  where  $OP$  stands for ordinary prime and  $GP$  for Gaussian prime. In order to prove this we need to do three steps. First, there are a total of four possibilities:  $(OP, OP)$ ;  $(GP, GP)$ ;  $(OP, GP)$  and  $(GP, OP)$ . Second, equate  $OP$  with  $4n + 1$  and  $GP$  with  $4n + 3$  where  $n$  is a positive integer. Third, test each possibility. Consider the first case or  $(OP, OP)$  then  $(p, p + 2) = (4n + 1, 4m + 1)$  where  $m > n$  otherwise the prime  $p + 2$  is not bigger than  $p$  as it must be. Solving for  $m$  we get  $m = n + \frac{1}{2}$  thus  $m$  is not an integer. Consequently, there is no twin prime that consists of a pair of ordinary primes. If you repeat this procedure for the remaining three cases it is easy to conclude that  $m$  is an integer only for  $(OP, GP)$  and  $(GP, OP)$  as expected. Hence, we have just proved that every twin prime is either a pair of an ordinary prime and a Gaussian prime or vice versa.

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## How can we find the prime factors of Fermat numbers?

In my book, I have conjectured that every prime factor of  $F(n)$  for  $n > 4$  can be obtained by executing the mathematical operation  $\gcd[F(n), b^{2^m} - 1]$  where the three letter  $\gcd$  stands for greatest common divisor and  $b$  is the base and  $m$  is the positive integer. For example, if we substitute  $b = 15409$  and  $m = 2$  into  $F(6)$  the prime factor 274177 is found. Surprisingly, the choice for  $(b, m)$  is not unique! In fact, for both  $(1071, 4)$  and  $(963, 8)$  we get the same prime factor. Please note that as the value of  $b$  decreases that of  $m$  increases and such is not a mere coincidence.