

High School

What are perfect numbers?

A perfect number equals the sum of all its divisors except the given number. For example, all the divisors of six except 6 are 1, 2 and 3. If we add these three integers, we get six. The next perfect number is 28.

What is a binary pattern?

In general, a binary pattern is any sequence of binary digits (zeros and ones) that form a pattern. For example, 10101010 is a binary pattern.

What is the twin prime conjecture?

The twin prime conjecture states that there is an infinitude of primes $(p, p + 2)$ where p is a prime such as $(5, 7)$, and so on. This is just one of the many unsolved problems in number theory.

What are Gödel's incompleteness theorems?

Kurt Gödel (1906 – 1978) was a logician who showed that mathematics has certain intrinsic limitations. His two incompleteness theorems demonstrated that any mathematical theory based on arithmetic cannot be both consistent and complete. In addition, there are statements in arithmetic which are true, but cannot be derived from the rules of arithmetic. As a result, there are mathematical problems that can never be solved!

What is meant by “mathematical physics”?

Prior to the discoveries of Isaac Newton (1642 – 1727) physics was a branch of philosophy. Mathematical physics was born with the formulation of his second law of motion, $F = ma$ where F stands for force, m for mass and a for acceleration. In other words, force equals mass times acceleration.

What is quantum physics?

While the laws of Newton apply to the macroscopic world, quantum physics describes the atom itself, that is, the microscopic realm.

What are the RSA challenge numbers?

RSA Security Inc. (www.rsa.com) was founded based on its well-known algorithm and periodically holds several world-wide competitions in the field of data encryption. The most famous of these consists in finding the prime factorization of any of the eight RSA challenge numbers posted in its website.

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What are Fermat numbers?

Fermat numbers are defined as numbers of the form $F(n) = 2^{2^n} + 1$ where n is a positive integer. Presently, mathematicians know that only the first five Fermat numbers are prime. For example, $F(0) = 3$ and $F(1) = 5$. After 65537, no more Fermat primes have been found.

What is set theory?

Georg Cantor (1845 – 1918), a German mathematician and philosopher, was the creator of set theory. It is a mathematical system that describes collections of real or abstract objects. Such objects are usually called elements. A set can have any number of elements. For example, the set of primes is infinite and so are the sets of both odd and even integers.

What are practical numbers?

A practical number is a positive integer such that all the integers smaller than itself can be represented as sums of different divisors of this number. Take the number 8 as an example; its divisors are 1, 2 and 4. The integers smaller than 8 are 1, 2, 3, 4, 5, 6, and 7. Thus, it is very easy to realize that $3 = 1 + 2$; $5 = 1 + 4$; $6 = 2 + 4$ and $7 = 1 + 2 + 4$ which are all divisors of 8. The first few practical numbers are 1, 2, 4, 6, 8, 12, 16 and so on.

What is a digital root?

The digital root is a mathematical operation defined as the repeated addition of all the decimal digits of a given number until the resulting sum is a single decimal digit. For example, the digital root of 13795 is $1 + 3 + 7 + 9 + 5 = 25$ and, once again, $2 + 5 = 7$. Hence, the digital root of 13795 is 7.

What are permutations?

Permutations are a special type of combination. To be more specific, the word “combination” represents the number of ways a given set of elements can be arranged such that each element is used only once. For example, given three letters, A , B , and C there a total of six ways you can arrange them such that each letter is used only once. These are ABC , ACB , BAC , BCA , CAB and CBA .

How can prime numbers be connected to the complex numbers?

By now we know what prime numbers are. On the other hand, complex numbers can be defined as $z = x + yi$ where x is its real part and y is the imaginary part. Please, note that i stands for the imaginary number $\sqrt{-1}$ or $i^2 = -1$. The mathematician Carl Friedrich Gauss (1777 – 1855) discovered that any number N that can be expressed as the sum of two squares must also be the product of two complex numbers. In mathematical terms, $N = x^2 + y^2 = (x + yi)(x - yi)$. For example, the prime number $5 = 1^2 + 2^2 = (1 + 2i)(1 - 2i)$ and so on.

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Is there a link between the Collatz problem and the Goldbach conjecture?

Yes, there is. The Collatz problem is defined by the function $f(x) = 3x + 1$ where x is a positive integer. Consider $x = p$ where p stands for prime, then $f(x)$ becomes $f(p) = 3p + 1$. Since every prime greater than two is an odd integer, it is clear that $3p + 1$ must be even. Therefore, we get that $3p + 1 = 2n$ where $n = 5, 8, 11, \text{ etc.}$, until infinity. If we rewrite the previous equation as $3p = 2n - 1$ its left-hand side consists of the addition of three identical primes while its right-hand side equals some odd integer. Such is equivalent to the statement $p + p + p = \text{odd integer}$ which is a special case of the Goldbach conjecture. In other words, every odd integer greater than five which is not a prime can be represented as the sum of three identical primes. For example, $3 + 3 + 3 = 9$ and so on.

What is elliptic curve cryptography?

Elliptic curve cryptography is an encryption-decryption scheme based on geometry, more specifically, elliptic curves.

What is fractal geometry?

The term “fractal” was coined by Benoit Mandelbrot (1924 – 2010) and roughly means fragmented and irregular. The most striking feature of this new type of geometry is the property of self-similarity where an object at ever scale is identical to the whole. Consider the familiar vegetable, cauliflower. If you detach any part of it, you will notice that the overall shape of the given fragment is similar to that of the whole vegetable.

What is meant by “fractal dimension”?

While Euclidean geometry concerns objects of integer dimensions (one, two or three), fractal geometry applies to non-integer dimensions or fractal dimensions such as 1.4 or 2.7, for example.

What is meant by the “least significant digit” of a number?

The least significant digit is simply the last decimal digit of the given number. For example, the least significant digit of 985641 is 1.

What is modular arithmetic?

Modular arithmetic is a mathematical operation. For example, $10 \bmod 3 = 1$ means the remainder when ten is divided by three or one.

What is trial division?

Trial division is a method which consists in dividing a composite number by 2, 3, 5, 7 and so on until some factor is found. For example, if we want to factor 2923, trial division is an efficient technique since $2923 = 37 \times 79$.